ARASU ENGINEERING COLLEGE, KUMBAKONAM

DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

Academic year 2018-2019

QUESTION BANK

CS6702-Graph Theory and Applications

UNIT I INTRODUCTION

PART – A

1. Define Graph. [CO1 LOT/K1]
2. Define Simple graph. [CO1 LOT/K1]
3. Write few problems solved by the applications of graph theory. [CO1 LOT/K1]
4. Define incidence, adjacent and degree. [CO1 LOT/K1]
5. What are finite and infinite graphs? [CO1 LOT/K1]
6. Define Isolated and pendent vertex. [CO1 LOT/K1]
7. Define null graph. [CO1 LOT/K1]
8. Define Multigraph. [CO1 LOT/K1]
9. Define complete graph. [CO1 LOT/K1]
10. Define Regular graph. [CO1 LOT/K1]
11. Define Cycles. [CO1 LOT/K1]
12. Define Isomorphism. [CO1 LOT/K1]
13. What is Sub graph? [CO1 LOT/K1]
14. Define Walk, Path and Circuit. [CO1 LOT/K1]
15. Define connected graph. What is connectedness? [CO1 LOT/K1]
16. Define Euler graph. Show that an Euler graph is connected except for any isolated vertex that graph may have. [CO1 LOT/K2]
17. Define Hamiltonian circuits and paths. [CO1 LOT/K1]
18. Define Tree. [CO1 LOT/K1]
19. List out few Properties of trees. [CO1 LOT/K1]
20. What is Distance in a tree? [CO1 LOT/K1]
21. Define eccentricity and center. [CO1 LOT/K1]

22. Define distance metric. [CO1 LOT/K1]
23. What are the Radius and Diameter in a tree? [CO1 LOT/K1]
24. Define rooted tree. [CO1 LOT/K1]
25. Define rooted binary tree. [CO1 LOT/K1]
26. Can there be path longer than a Hamiltonian path in a simple, connected, undirected graph? Why? [CO1 LOT/K2]
27. Determine the number of vertices for graph G, which has 15 edges and each vertex has degree 6.Is the graph G be simple graph? [CO1 LOT/K3]
28. Suppose G is finite cycle free connected graph with at least one edge. Show that G has at least two vertices of degree 1. [CO1 LOT/K3]
29. For which of the following does there exist a simple graph G= (V,E) satisfying the specified conditions? [CO1 HOT/K4]
    a. It as 3 components 20 vertices and 16 edges
    b. It is connected and has 10 edges 5 vertices and fewer than 6 cycles
    c. It has 7 vertices, 10 edges and more than two components
30. The maximum degree of any vertex in simple graph with n vertices is n-1. Give reasons. [CO1 LOT/K2]
31. Verify that two graphs a and b in the following figure are isomorphic. Mention the reason for it. [CO1 LOT/K3]

PART – B

1. Explain various applications of graph. [CO1 HOT/K5]
2. Define the following terms: [CO1 LOT/K1]
   i. Walk
   ii. Euler path
   iii. Hamiltonian path
   iv. Sub graph
   v. Circuit
   vi. Complete Graph

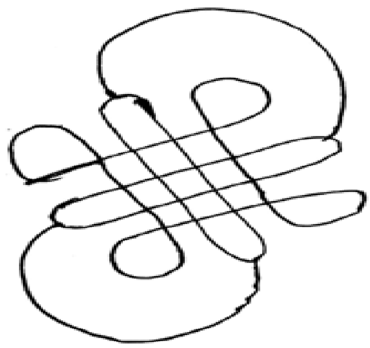
   From the given graph draw the following: [CO1 HOT/K4]

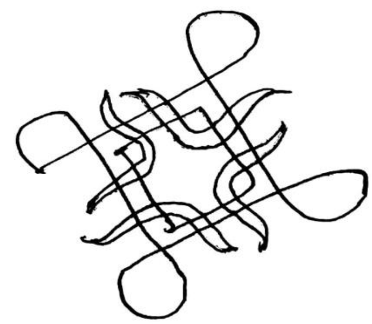
   vii. Walk of length 6
   viii. Is this an Euler Graph? Give reasons
   ix. Is there a Hamiltonian path for this graph? Give reasons.
   x. Find at least two complete sub graphs.

3. Show that a connected graph G is an Euler graph if all vertices are even degree. [CO1 HOT/K5]
4. Prove that a simple graph with n vertices and k components can have at most (n-k)(n-k+1)/2 edges. [CO1 HOT/K5]

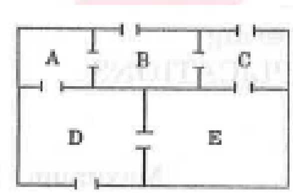
5. Are they isomorphic? [CO1 LOT/K3]
6. Prove that in a complete graph with n vertices there are (n-1)/2 edges-disjoint Hamiltonian circuits, if n is odd number ≥3. [CO1 HOT/K5]
7. Prove the given statement, “A tree with n vertices has n-1 edges”. [CO1 HOT/K5]
8. Prove that, any connected graph with n vertices has n-1 edges is a tree. [CO1 HOT/K5]
9. Show that a graph is a tree if and only if it is minimally connected. [CO1 HOT/K5]
10. Prove that, a graph G with n vertices has n-1 edges and no circuits are connected. [CO1 HOT/K5]
11. State and prove Dirac’s Theorem. [CO1 HOT/K5]
12. Draw a graph isomorphic to given graph G such that no edge is crossing others. [CO1 HOT/K6]

13. Also prove that every tree has one or more centers. [CO1 HOT/K5]
14. Can the kolams given in the following figures be drawn with out lifting your hands and not overdrawing any part of the kolam? Substantiate your answer with graph theory knowledge. If not possible, make it possible by adding some curves. [CO1 HOT/K6]



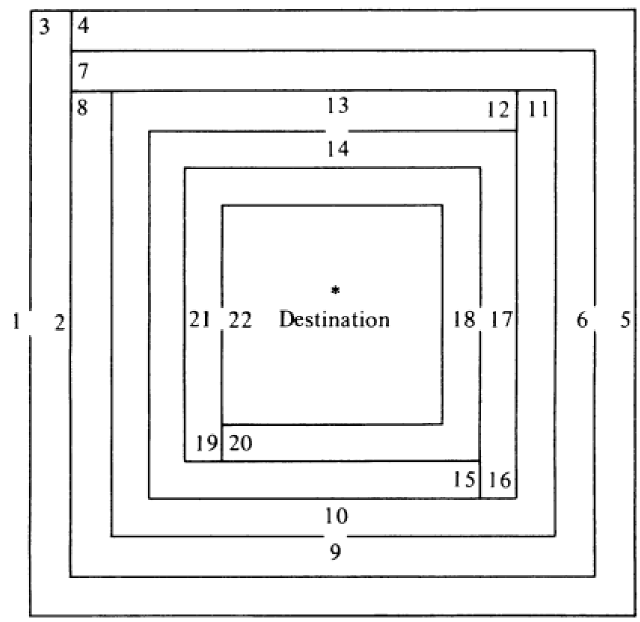


15. Seven children in a street play a game in circuit arrangement. If no child holds hand with the same playmate twice, how many times can these arrangements possible? Write all possible arrangements. [CO1 HOT/K4]
16. Prove that if a graph has exactly two vertices of odd degree, there must be path joining these vertices. [CO1 HOT/K5]
17. Prove that any two simple connected graphs with n vertices, all of degree two is isomorphic. [CO1 HOT/K5]
18. State four properties of tree and prove them. [CO1 LOT/K1]
19. Prove that in any tree, there are at least two pendant vertexes. [CO1 HOT/K5]
20. The following represents a floor plan with doors between the rooms and outside indicated. The real estate agent would like to tour the house, starting and ending outside, by going through each door exactly once. What is the fewest number doors that should be added and where they should be placed in order to make this tour possible? Give reasons for your answer.(6) [CO1 HOT/K6]



21. Write down adjacency and incidence matrix for the following graph. [CO1 HOT/K4]
22. Show that Hamiltonian path is the spanning tree. [CO1 HOT/K5]

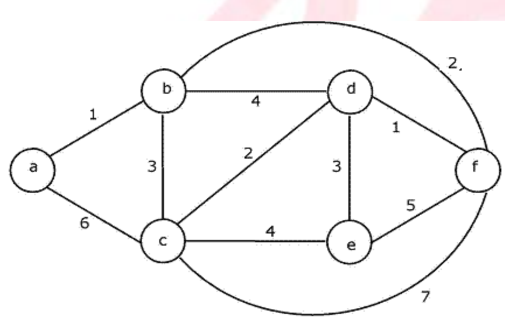
23. Given a maze as shown in following figure, represent this maze by means of graph such that a vertex represents a corridor or dead end. An edge represents both between two vertices and find the path to destination. [CO1 HOT/K6]



UNIT II TREES, CONNECTIVITY & PLANARITY

PART – A

1. Define Spanning trees. [CO2 LOT/K1]
2. Define Branch and chord. [CO2 LOT/K1]
3. Define complement of tree. [CO2 LOT/K1]
4. Define Rank and Nullity. [CO2 LOT/K1]
5. How Fundamental circuits created? [CO2 LOT/K1]
6. Define Spanning trees in a weighted graph. [CO2 LOT/K1]
7. Define degree-constrained shortest spanning tree. [CO2 LOT/K1]
8. Define cut sets and give example. [CO2 LOT/K1]
9. Write the Properties of cut set. [CO2 LOT/K1]
10. Define Fundamental circuits. [CO2 LOT/K1]
11. Define Fundamental cut sets. [CO2 LOT/K1]
12. Define edge connectivity. [CO2 LOT/K1]
13. Define vertex connectivity. [CO2 LOT/K1]
14. Define separable and non-separable graph. [CO2 LOT/K1]
15. Define articulation point. [CO2 LOT/K1]
16. What is network flows? [CO2 LOT/K1]
17. Define max-flow and min-cut theorem (equation). [CO2 LOT/K1]
18. Define component (or block) of graph. [CO2 LOT/K1]
19. Define 1-Isomorphism. [CO2 LOT/K1]
20. Define 2-Isomorphism. [CO2 LOT/K1]
21. Briefly explain combinational and geometric graphs. [CO2 LOT/K1]
22. Distinguish between Planar and non-planar graphs. [CO2 LOT/K1]
23. Define embedding graph. [CO2 LOT/K1]
24. Define region in graph. [CO2 LOT/K1]
25. Why the graph is embedding on sphere? [CO2 LOT/K1]
26. What are the applications of planar graph? [CO2 LOT/K1]
27. Define planar graph and give example. [CO2 LOT/K1]
28. Identify two spanning tree for the following graph. [CO2, HOT/K4]



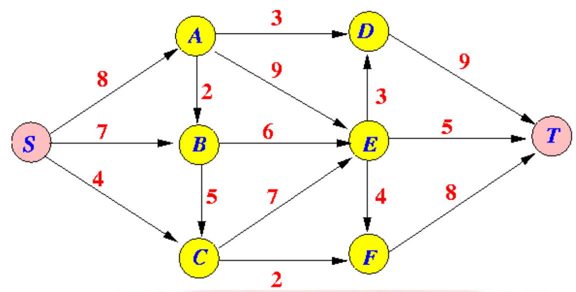
29. In a tree every vertex is the cut-vertex justify the claim. [CO2 HOT/K4]
30. A simple planar graph to which no edge can be added without destroying its planarity is a maximal planar graph. Prove that every region in maximal planar graph is triangle. [CO2 HOT/K4]
31. State Kuratowski's Theorem. [CO2 LOT/K1]
32. Define homeomorphic graphs and give examples. [CO2 LOT/K1]

PART – B

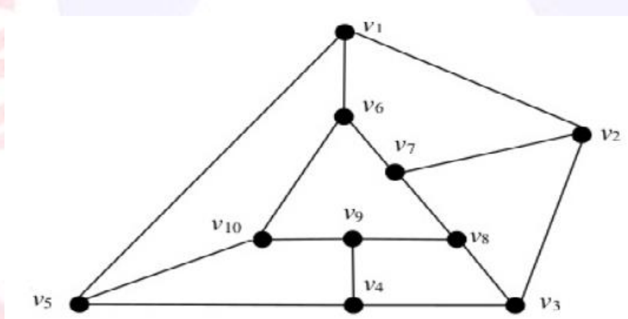
1. Find the shortest spanning tree for the following graph. [CO2 HOT/K6]

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 \end{array} \\ \left[\begin{array}{cccccc} - & 10 & 16 & 11 & 10 & 17 \\ 10 & - & 9.5 & \infty & \infty & 19.5 \\ 16 & 9\ 5 & - & 7 & \infty & 12 \\ 11 & \infty & 7 & - & 8 & 7 \\ 10 & \infty & \infty & 8 & - & 9 \\ 17 & 19.5 & 12 & 7 & 9 & - \end{array}\right] \end{array}$$

2. Explain 1 - isomorphism and 2 - isomorphism of graphs with your own example. [CO2 HOT/K5]
3. Prove that a connected graph G with *n* vertices and *e* edges has *e*-*n*+*2* regions. [CO2 HOT/K6]
4. Write all possible spanning trees for K5. [CO2 HOT/K6]
5. Prove that every cut-set in a connected graph G must contain at least one branch of every spanning tree of G. [CO2 HOT/K5]
6. Prove that the every circuit which has even number of edges in common with any cut-set. [CO2 HOT/K5]
7. Show that the ring sum of any two cut-sets in a graph is either a third cut set or en edge disjoint union of cut sets. [CO2 HOT/K5]
8. Explain network flow problem in detail. [CO2 HOT/K5]
9. If G_1 and G_2 are two 1-isomorphic graphs, the rank of G_1 equals the rank of G_2 and the nullity of G_1 equals the nullity of G_2, prove this. [CO2 HOT/K5]
10. Prove that any two graphs are 2-isomorphic if and only if they have circuit correspondence. [CO2 HOT/K5]
11. Explain max-flow min-cut theorem. Using it calculate the maximum flow between the nodes D and E in the given graph. The number on the line represents capacity. [CO2 HOT/K5]



12. Explain about fundamental cut-set and fundamental circuit in a graph. [CO2 HOT/K5]
13. Prove that every connected graph has at least one spanning tree. [CO2 HOT/K5]
14. Prove the graphs K_5 and $K_{3,3}$ are non-planar. [CO2 HOT/K5]
15. Define spanning tree and give example. [CO2 LOT/K1]
16. A farm has six walled plots full of water. The graph representation of it is given below. Use the concepts of spanning tree, cut sets appropriately to determine the following:
    a. How many walls will have to be broken so that all the water can be drained out?
    b. If only one plot was full of water and this had to be drained into all other plots, then how many walls need to be broken? [CO2 HOT/K4]



17. State the Euler's formula relating the number of vertices, edges and faces of a planar connected graph. Give two conditions for testing for planarity of a given graph. Give a sample graph that is planar and another that is non-planar. [CO2 HOT/K5]
18. Show that starting from any spanning tree of graph G, every other spanning tree of graph G can be obtained by successive cyclic interchanges. [CO2 HOT/K5]
19. Define vertex connectivity and edge connectivity. Give the relation between them. [CO2 LOT/K1]
20. Show by drawing the graphs, that two graphs with the same rank and the same nullity need not to be 2-isomorphic. [CO2 HOT/K5]
21. State Kuratowski's Theorem and use it in order to prove the graph in following graph is non-planar. [CO2 HOT/K5]

22. Define 2-Isomorphism and prove that the rank and nullity of graph are invariant under 2-Isomorphism. [CO2 HOT/K5]
23. Prove that
    With respect to given spanning tree T, a branch bi that determines a fundamental cut-set S is contained in every circuit associated with the chords in S and no others.
    Prove that distance between any two spanning tree is metric.
    Find two different minimum spanning trees of a graph with v={1,2,3,4} is described by[CO2 HOT/K5]

    $\varphi = \begin{pmatrix} a & b & c & d & e & f \\ \{1, 2\} & \{1, 2\} & \{1, 4\} & \{2,3\} & \{3, 4\} & \{3, 4\} \end{pmatrix}$. It has weights on its edges given by $\lambda = \begin{pmatrix} a & b & c & d & e & f \\ 3 & 2 & 1 & 2 & 4 & 2 \end{pmatrix}$.

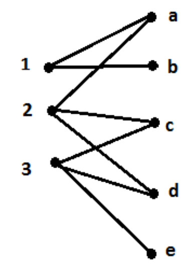
24. Prove that Euler graph cannot have a cut-set with odd number of edges. [CO2 HOT/K5]
25. Construct a graph G with following properties. Edge connectivity of G=4, vertex connectivity of G=3 and degree of every vertex G>=5. [CO2 HOT/K6]
26. Derive the formula for the number of regions in planar graph, G with n vertices and e edges. Also prove that planar graph with triangle region can atmost have (3n-6) edges. [CO2 HOT/K5]
27. List the properties of cut-set. [CO2 LOT/K1]

UNIT III MATRICES, COLOURING AND DIRECTED GRAPH

PART – A

1. What is proper coloring? [CO3 LOT/K1]
2. Define chromatic number. [CO3 LOT/K1]
3. Write the properties of chromatic numbers (observations). [CO3 LOT/K1]
4. Define chromatic partitioning. [CO3 LOT/K1]
5. Define independent set and maximal independent set. [CO3 LOT/K1]
6. Define uniquely colorable graph. [CO3 LOT/K1]
7. Define dominating set and minimal dominating set. [CO3 LOT/K1]
8. Define chromatic polynomial. [CO3 LOT/K1]
9. Define matching (Assignment). [CO3 LOT/K1]
10. What is covering? [CO3 LOT/K1]
11. Define minimal cover. [CO3 LOT/K1]
12. What is dimer covering? [CO3 LOT/K1]
13. Define four color problem / conjecture. [CO3 LOT/K1]
14. State five color theorem. [CO3 LOT/K1]
15. Write about vertex coloring and region coloring. [CO3 LOT/K1]
16. Define Directed graphs. [CO3 LOT/K1]
17. Define isomorphic digraph. [CO3 LOT/K1]
18. List out some types of directed graphs. [CO3 LOT/K1]
19. Define simple digraphs. [CO3 LOT/K1]
20. Define asymmetric digraphs (Anti-symmetric). [CO3 LOT/K1]
21. What is meant by symmetric digraphs? [CO3 LOT/K1]
22. Define simple symmetric digraphs. [CO3 LOT/K1]
23. Define simple asymmetric digraphs. [CO3 LOT/K1]
24. Give example for complete digraphs. [CO3 LOT/K1]
25. Define complete symmetric digraphs. [CO3 LOT/K1]
26. Define complete asymmetric digraphs (tournament). [CO3 LOT/K1]
27. Define balance digraph (a pseudo symmetric digraph or an isograph). [CO3 LOT/K1]
28. Define binary relations. [CO3 LOT/K1]
29. What is directed path? [CO3 LOT/K1]
30. Write the types of connected digraphs. [CO3 LOT/K1]
31. Prove that the graph of n vertices is complete if and only if its chromatic polynomial is
    $$P_n(\lambda) = \lambda(\lambda - 1)(\lambda - 2) \dots (\lambda - n + 1).$$ [CO3 HOT/K5]
32. Define the two types of connectedness in graph with example. [CO3 LOT/K1]
33. Find the chromatic number of complete graph with n vertices. [CO3 LOT/K3]
34. Draw k8 and k9 and show that thickness of k8 is 2 while thickness of k9 is 3. [CO3 HOT/K6]
35. Let graph G is 2-chromatic then prove it is bipartite. [CO3 HOT/K5]
36. What is meant by regularization of planar graph? Give example. [CO3 LOT/K1]

37. Give an example of transitive relation. [CO3 LOT/K1]
38. Does the following graph have maximal matching? Give reasons. [CO3 HOT/K4]

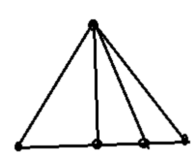


PART – B

1. Define chromatic polynomial. Find the chromatic polynomial for the following graph. [CO3 HOT/K4]

2. Explain matching and bipartite graph in detail. [CO3 HOT/K5]
3. Write the observations of minimal covering of a graph. [CO3 LOT/K1]
4. Prove that the vertices of every planar graph can be properly colored with five colors.(8) [CO3 HOT/K5]
5. State and prove five color theorem. [CO3 HOT/K5]
6. Illustrate four-color problem. (5) [CO3 LOT/K2]
7. Explain Euler digraphs in detail. [CO3 HOT/K5]
8. Obtain the chromatic polynomial of the given graph using the theorem. (7) [CO3 HOT/K4]

$$\mathrm{Pn}(\lambda) \text{ of } G = \mathrm{Pn}(\lambda) \text{ of } G' + \mathrm{Pn}(\lambda) \text{ of } G''.$$



9. Define the following and give one example to each:(8) [CO3 HOT/K3]
    - Complete Matching
    - Minimal Covering
    - Balanced Digraph
    - Strongly Connected Digraph
    - Fragment in Digraph
    - Complete Symmetric Graph
    - Equivalence Graph
    - Accessibility in digraph
10. Prove that a digraph G is an Euler digraph if and only if G is connected and is balanced. Draw an example Euler digraph of 6 vertices. [CO3 HOT/K5]
11. Prove that every tree with two or more vertices is two chromatic. [CO3 HOT/K5]
12. Prove that a covering g of graph G is minimal iff g contains no path of length three or

more. [CO3 HOT/K5]

13. Discuss about four types of digraphs with suitable example. [CO3 HOT/K5]
14. Describe the steps to find adjacency matrix and incidence matrix for directed graph with suitable example. [CO3 LOT/K3]
15. Write a note on chromatic polynomial and its applications. [CO3 LOT/K1]
16. If G is a tree with n vertices, then prove that its chromatic polynomial is[CO3 HOT/K5]
    $$P_n(\lambda) = \lambda\,(\lambda - 1)^{n-1}.$$
17. Define chromatic number. Prove that a graph with at least one edge is 2-chromotic if and only if it has no circuits of odd length. [CO3 HOT/K5]
18. Discuss complete matching and minimal covering in graph G. Give one application example to each. [CO3 HOT/K5]
19. When is a digraph an Euler digraph? Draw an Euler digraph. [CO3 HOT/K4]
20. How will you find all maximal independent set? Explain. [CO3 LOT/K1]
21. How covering of graph is verified? Discuss about it. [CO3 HOT/K5]

UNIT IV PERMUTATIONS & COMBINATIONS
PART – A

1. Define Fundamental principles of counting.
2. Define rule of sum.
3. Define rule of Product
4. Define Permutations
5. Define combinations
6. State Binomial theorem
7. Define combinations with repetition
8. Define Catalan numbers
9. Write the Principle of inclusion and exclusion formula.
10. Define Derangements
11. What is meant by Arrangements with forbidden (banned) positions.
12. Find the number of permutations in the word COMPUTER if only five of the letters are used.
13. Find the number of arrangements of four letters in BALL.
14. Find all the permutations for the letters a,c,t.
15. Show how many permutations are there for the eight letters a, c, f, g, i, t, w, x.
16. A student taking a history examination is directed to answer any seven of 10 essay questions. There is no concern about order here. Explain how many ways the student can answer the examination.
17. Find the Binomial coefficient of $x^5 y^2$ in $(x+y)^7$
18. Find the number of de-arrangements of 1,2,3,4
19. How many words can be formed by using all letters of the word 'BIHAR'?
20. How many arrangements can be made out of the letters of the word 'ENGINEERING' ?
21. In how many possible ways could a student answer a 10 question true-false test?
22. How many permutations of size 3 can one produce with the letters m, r, a, f and t?
23. In how many ways the letters of the word ‘APPLE’ is arranged?
24. In how many ways can a group of 5 men and 2 women be made out of a total of 7 men and 3 women?
25. Find the number of positive integers n, 1≤n≤500, that are not divisible by 5.
26. Find the number of non-negative integral solutions to X1+X2+X3+X4=20.
    Find the number of ways letters in TRIANGEL can be arranged such that vowels occur together.
27. THALASSEMIA is genetic blood disorder. How many ways can the letter in THALASSEMIA be arranged so that all three A’s together?

Part B

1. Determine the number of six-digit integers (no leading zeros) in which (a) no digits may be repeated; (b) digits may be repeated; (c) answer (a) and (b) with the extra conditions that the six-digit integer is (i) odd; (ii) divisible by 10; (iii) divisible by 4.
2. Determine the co-efficient x9y6 in the expansion (4y-x)15.
3. Find the value n for the following 2P(n,2)+50=P(2n,2)
4. How many arrangements of the letters in MISSISSIPPI have no consecutive S's?
5. How many arrangements are there of all the vowels adjacent in SOCIOLOGICAL?
6. How many distinct four-digit integers can one make from digits 1 ,3 ,3, 7, 7 and 8?
7. A survey of 150 college students reveals that: 83 own Cars, 97 own Bikes, 28 own Motorcycles, 53 own a car and a bike, 14 own a car and a motorcycle, 7 own a bike and a motorcycle, 2 own all three.
8. In a survey of chewing gum tastes of a group of baseball players, it was found that 22 liked juicy fruit, 25 liked spearmint, 39 liked bubble gum, 9 liked both spearmint and juicy fruit, 17 liked juicy fruit and bubble gum, 20 liked spearmint and bubble gum, 6 liked all three and 4 liked none of these. How many base ball players are surveyed?
9. In a group of 6 boys and 4 girls, four children are to be selected. In how many different ways can they be selected such that at least one boy should be there?
10. There are six students in a group and their roll numbers are, S1, S2, S3, S4, S5 and S6. They are given with six assignments numbered 1 to 6. Each has to solve one assignment. How many ways the arrangements can be distributed such that a student is not getting assignment number same as his roll number?
11. How many permutations of size 3 can one produce with the letters m, r, a, f and t?
12. In how many ways can the 26 letters of alphabet be permitted so that none of the patterns car, dog, pun or byte occurs?
13. In how many ways can three boys be seated on five chairs?
14. Five professor P1,P2,P3,P4,P5 are to be made class advisor for five sections C1,C2,C3,C4,c5 one professor for each section.p1 and p2 do not wish to become the class advisors for C1 or C2,P3 and P4 for C4 and P5 for C3 or C4 or

C5.in how many ways can the professors be assigned the work(without displaying any professor)? (8)

15. Using the principle inclusion and exclusion find the number of prime numbers not exceeding 100
16. A gym coach must select 11 seniors to play on a football team. If he can make his selection in 12,376 ways, how many seniors are eligible to play? (6)
17. Rama has two dozen each of n different colored beads. If she can select 20 beads (with repetitions of colors allowed), in 230,230 ways, what is the value of n? (7)
18. At Flo's Flower Shop, Flo wants to arrange 15 different plants on five shelves for a window display. In how many ways can she arrange them so that each shelf has at least one, but no more than four, plants?
19. How many ways the prize of Rs.1000 prize can be distributed among 4 peoples and the prize can be divided only in units of 100? (8)

    a)No restrictions

    b)Each should get minimum of 100

    c)Each should get minimum of 200
20. How many arrangements are there of all the vowels not adjacent in DRIVER? (5)
21. Determine the number of (staircase) paths in the xy-plane from (2, 1) to (7, 4), where each path is made up of individual steps going 1 unit to the right (R) or one unit upward (U). (7)
22. How many times the print statement executed in this program segment?

```
for i := 1 to 20 do
  for j := 1 to i do
    for k := 1 to j do
      print (i * j + k)
```

(6)

23. How many integer solutions are possible for $X_1 + X_2 + X_3 + X_4 + X_5 < 40$ where $x_i \geq -3$, $i \leq i \leq 5$ (5)
24. How many integers between 1 and 300 (inc.) are not divisible by at least one of 5, 6, 8?

UNIT V GENERATING FUNCTIONS

PART A

1. Define recurrence relation.
2. Define generating function.
3. Define Exponential generating function.
4. What is Partitions of integer?
5. Define Maclaurin series expansion of e^x and e^{-x}.
6. Define Summation operator
7. Define First order linear recurrence relation
8. Define Second order recurrence relation
9. Briefly explain Non-homogeneous recurrence relation.
10. Find the generating function for the sequence 3,-3,3,-3,…
11. If the sequence an = 3.2^n ,n≥1 , then find the corresponding recurrence relation.
12. Find the recurrence relation for $s(n) = 6(-5)^n$.
13. Find the generating function for the sequence S with terms 1,2,3,4,…
14. What is the generating function for the sequence 1,1,1,1,1?
15. Find the recurrence relation for the Fibonacci sequence.
16. Determine the generating function for the number of ways to distribute 35 pennies (from an unlimited supply) among five children if (a) there are no restrictions; (b) each child gets at least 10; (c) each child gets at least 20; (d) the oldest child gets at least 100; and (e) the two youngest children must each get at least 100
17. In how many ways can 3000 identical envelopes be divided, in packages of 25, among four student groups so that each group gets at least 150, but not more than 1000, of the envelopes?
18. If a fair die is rolled 12 times, what is the probability that the sum of the rolls is 30?
19. Show that $(1 - 4x)^{-1/2}$ generates the sequence $\binom{2n}{n}$, $n \in \mathbb{N}$.
20. What is the generating function for the sequence 1, 1, 0, 1, 1, 1, . and 1, 1, 1, 3, 1, 1, . .

21. Find the generating function for the sequence 0,2,6,12,20,30,42
22. Verify that for all *n* Z+

$$\binom{2n}{n} = \sum_{i=0}^{n} \binom{n}{i}^2$$

23. Find the coefficient of x5 in (1 - 2x)-7
24. Solve the recurrence relation an = 7an-1 where n≥1 and a2=98.
25. A company hires 11 new employees, each of whom is to be assigned to one of four subdivisions. Each subdivision will get at least one new employee. In how many ways can these assignments be made?
26. Give explanation for the following: Generating function for the no. of ways to have n cents in pennies and nickels =(1+x+x2+...)(1+x5+x10+...)
27. Solve the recurrence relation an+1 – an = 3n2 – n, n>=0, a0=3

PART B

1. While shopping one Saturday, Mildred buys 12 oranges for her children, Grace, Mary, and Frank. In how many ways can she distribute the oranges so that Grace gets at least four,and Mary and Frank get at least two, but Frank gets no more than five?

2. If there is an unlimited number (or at least 24 of each color) of red, green, white, and black jelly beans, in how many ways can Douglas select 24 of these candies so that he has an even number of white beans and at least six black ones?

3. Find the generating function for the sequence $\binom{-n}{0}, \binom{-n}{1}, \binom{-n}{2}, \binom{-n}{3}, \ldots$

4. There is one such partition of the integer 1 —namely, 1— but there are no such partitions of the integer 2. For the integer 3 we have two of these partitions: 3 and 1 + 1 +

1. When we examine the possibilities for the integer 4, we find the one partition 3 + 1.

5. A ship carries 48 flags, 12 each of the colors red, white, blue, and black. Twelve of these

flags are placed on a vertical pole in order to communicate a signal to other ships.

a) How many of these signals use an even number of blue flags and an odd number of black

flags?

b) How many of the signals have at least three white flags or no white flags at all? In this

situation we use the exponential generating function.

6. Solve the recurrence relation an + an- 1 — 6an-2 = 0, where n ≥ 2 and ao = -1,a1 = 8.

7. Solve the recurrence relation of the fibonacci sequence of number

$$f_n = f_{n-1} + f_{n-2}, \quad n > 2 \quad \text{with initial condition , } f_1 = 1, \, and \; f_2 = 1.$$

8. In many programming languages one may consider those legal arithmetic expressions,

without parentheses, that are made up of the digits 0, 1, 2, , , 9 and the binary operation

symbols +, *, /. For example, 3 + 4 and 2 + 3 * 5 are legal arithmetic expressions; 8 +

*9 is not. Here 2 + 3 * 5 = 17, since there is a hierarchy of operations: Multiplication

and division are performed before addition. Operations at the same level are performed in

their order of appearance as the expression is scanned from left to right. Determine the

recurrence relation for an.

9. Suppose we have a 2 x n chessboard, for n E Z+. The case for n = 4 is shown in part (a)

of Fig.10.5. We wish to cover such a chessboard using 2 x 1 (vertical) dominoes, which

can also be used as 1 x 2 (horizontal) dominoes. Such dominoes (or tiles) are shown in

part (b) of Fig.10.5. Find the recurrence relation .

10. Solve the recurrence relation an+2-5an+1+6an using the generating functions.

11. Solve the relation an-3an-1 = n, n≥1, a0=1.

12. (i) Discuss about exponential generating function with an example.(10)

ii) Find the unique solution of the recurrence relation.6an-7an-1 = 0, n 1, a3 = 343.(6)

13. (i) The population of Mumbai city is 6,000,000 at the end of the year 2015. The number of immigrants is 20000n at the end of year n. the population of the city increases at the rate of 5% per year. Use a recurrence relation to determine the population of the city at the end of 2025. (8) (ii) Write short note on Summation operator. (8)

14. If an is count of number of ways a sequence of 1s and 2s will sum to n, for n>=0.Eg a3= 3. (i) 1, 1, 1; (ii) 1, 2, and (iii) 2, 1 sum up to 3. Find and solve a sequence relation for an. (16) 15. What are Ferrers diagrams? Describe how they are used to (i) represent integer partition (ii) Conjugate diagram or dual partitions (iii) self-conjugates (iv) representing bisections of two partition.(16)

16. Solve the recurrence relation an+2-4an+1+3an=-200 with a0=3000 and a1=3300.

17. Solve the Fibonacci relation Fn = Fn-1+Fn-2. 18. Find the recurrence relation from the sequence 0, 2, 6, 12, 20, 30, 42, … .